Confidence Regions and Approximated Multimodal Distributions

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Abstract

When economic models have potentially multiple equilibria, similar parameter values can lead to very different outcomes. Analogously, similar observations can be consistent with very different parameter values. Inference in these cases is potentially fragile. Fragile inference can also occur in standard estimation techniques like Generalized Method of Moments (GMM) or Maximum Likelihood (ML) when distinct sets with plausible parameter values are separated by implausible or even impossible parameter value. Fragility of inference can manifest itself in multi-modality of the estimator's density function. Standard techniques leading to asymptotic normal approximations cannot capture such bimodality. Saddlepoint approximations and Barndorff-Nielsen's celebrated p^*-formula on the other hand can capture multimodality. In this paper we show that in general this requires a crucial adjustment to the basic formula. The adjustment is based on a simple idea and is straightforward to implement, yet delivers important improvements. It is based on recognizing that certain outcomes are theoretically impossible and the density of the MLE should then equal zero, rather than the positive density that a straight application of p^* would suggest. This has implications for inference and we show how to use the new p^{**}-formula to construct improved confidence regions. These can be disjoint as a consequence of the bimodality. The degree of bimodality depends heavily on the value of an approximate ancillary statistic and conditioning on the observed value of this statistic is therefore desirable. The p^{**}-formula naturally delivers the relevant conditional distribution. We illustrate these results in small and large samples using a simple nonlinear regression model, a spatial correlation model networks, and an errors in variables model where the measurement errors in dependent and explanatory variables are correlated and allow for weak proxies.

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