Welfare effects – PAYG vs funded pension systems

Wencke Böhm
Bielefeld University
BiGSEM – Bielefeld Graduate School of Economics and Management
wboehm@wiwi.uni-bielefeld.de

Abstract

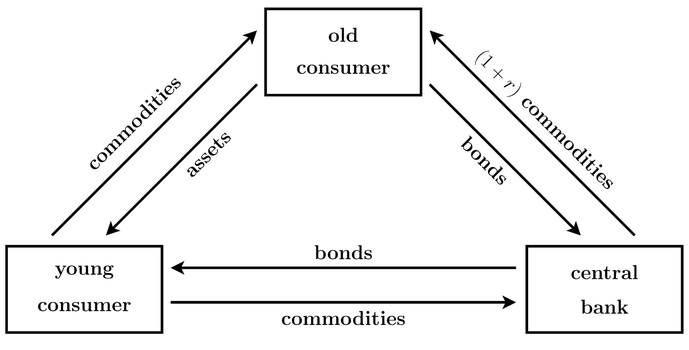
The aim of this paper is to analyze the implication of an introduction of a pension system (either PAYG or funded) on the welfare of the existing consumers, if initially no old-age pension system exists. According to the ongoing discussion in many western countries about the "right" pension system, we will compare a pay-as-you-go pension with a funded pension system in a typical CAPM setting and investigate the effects of heterogenous beliefs (consumer vs. pension system) on the price process of the risky assets. As in the most countries a pension system already exists, we will show which impact an increase of the tax rate has on the welfare of the economy.

1. Motivation

Observations in many western countries show a growing number of old people opposite to a vanishing number of young people, who are supposed to be responsible for the old-age pension payment. Driven by the ongoing demographic change we face a gap in financing old-age consumption through the existing pay-as-you-go pension system. One result of the public debate running in Germany is that consumers have to save privately in form of a funded pension system (Riester Rente) to ensure their wealth when old. It is stated that a funded in contrast to a PAYG pension system is not welfare-reducing. We will analyze this question in a setting, where our consumer and the funded system also are able to save in form of a risk-free asset, but can also invest in risky assets.

2. The Model

Given a two-period OLG-Model, where the economy consists of three market partipants: The Consumer (with mass normalized to one in every period), one pension system (PAYG or funded) and one central bank (which lives forever). The consumer has an initial endowment e consisting of a non-storeable commodity with a price normalized to one. He consumes only in his second period of life, hence when he is old, and receives no endowment or payment. He has to pay taxes $\tau\, e$ to the pension system, which will give the old-age pensioneers a pension payment in form of commodities. The consumer has to transfer, beside the pension payment, his commodities to the period of retirement. He can save in form of risk-free treasury bonds (y) or he can invest in risky assets (x). These will pay $p_{t+1} + d_{t+1}$ in the future period. p_{t+1} is tomorrow's equilibrium price and d_{t+1} is a dividend payment, described through a stationary Markov process. Treasury bonds are emitted by the central bank and can be bought or short-sold by the young consumer (and the funded pension system). They symbolize a vested right for future commodities and deliver $1 + r = R$ commodities tomorrow. Risky assets are sold only by the old consumer (and the funded pension system) to the young consumer (and the funded pension system). The following figure tries to clearify the structure of trade without a pension system and with no short-selling.



3. Pay-As-You-Go Pension System

The utility function of a consumer is given by

$$U(\mu, \sigma) = \frac{1}{1-\beta}\mu^{1-\beta} - \alpha\, \sigma^{1+\eta}$$

facing his budget constraint

$$(1-\tau)\, e = y_t + p_t\, x_t.$$

A consumer can invest in a portfolio consisting of risky assets and risk-free treasury bonds, which pay $1+r$ future commodities in the second period. Hence his decision problem can be characterized by

$$\varphi(e, \pi, V) := \underset{x \in \mathbb{R}^K}{arg\max}\; U\left(R_t\,(1-\tau)\, e + \pi_t\, x_t + R_{ps,t}\,\tau\, e, (x_t^2\, V)^{1/2}\right)$$
$$\Rightarrow x_t^* := \varphi(e, \pi_t, V) = \frac{\pi_t}{\alpha^2 V} - \frac{1}{\pi_t}\left((1-\tau)\; Re + R_{ps,t}\tau e\right)$$

$\pi_t = q_t^e - R\, p_t$, with q_t^e denoting consumer's current expectations for future cum-dividend prices. The market for risky assets has to be cleared in every period. Therefore we receive the following equilibrium price

$$p_t = \frac{2q_t^e - \alpha^2 V \bar{x}}{2R} + \frac{1}{R}\sqrt{\frac{1}{4}(\alpha^2 V \bar{x})^2 + \alpha^2 V\left((1-\tau)\, Re + R_{ps,t}\tau e\right)}.$$

The aim of the paper is to investigate the effects of an introduction of a pension system on the welfare of the consumer. We use aggregate consumption as a measure of welfare.

$$\hat{C}_t = R\, e + (p_t + d_t - R\, p_{t-1})\, \bar{x} + (1-R)\, \tau e$$

If a PAYG pension system exists, it is the best to have a very low tax rate τ, as the welfare is reduced by higher tax rates, i.e. the existence of a PAYG system is welfare reducing.

$$\frac{\partial \hat{C}_t}{\partial \tau} < 0 \qquad \text{if} \qquad 0 < R < \frac{1}{2} + \sqrt{\frac{1}{4} + 1 - \beta}$$

Theorem 3.1 *The introduction of a pay-as-you-go pension system will reduce (increase) the welfare of the consumer, if the return R of the treasury bond is higher (lower) than one (= $R_{ps,t}$).*

$$\begin{aligned} \hat{C}_t < C_t \quad &\textit{if} \quad R > 1\ \forall t \\ \hat{C}_t = C_t \quad &\textit{if} \quad R = 1\ \forall t \\ \hat{C}_t > C_t \quad &\textit{if} \quad R < 1\ \forall t \end{aligned}$$

4. Funded Pension System

Now the utility function of the consumer has a slightly different look, due to the risk his investment in the funded pension system captures.

$$U(\mu, \sigma, \sigma_{ps}) = \frac{1}{1-\beta}\mu^{1-\beta} - \alpha\, \sigma^{1+\eta} - \alpha\, \sigma_{ps}^{1+\eta}$$

Investment in the funded pension system is connected to some risk, which has to be taken into account by the consumer.

$$\begin{aligned} \varphi(e, \pi_t, V) \;:=\; & \underset{x \in \mathbb{R}^K}{arg\max}\; U\left(R_t\,(1-\tau)\, e + \pi_t\, x_t + R_{ps,t}\,\tau\, e, (x_t^2\, V)^{1/2}, (\tilde{x}_t^2\, V)^{1/2}\right) \\ \text{s.t. } & \left(\frac{\pi_t^e}{\alpha^2 V} - \frac{1}{\pi_t^e}\left((1-\tau)\; Re + R_{ps,t}\tau e\right)\right)\sqrt{V} + \tau e\, \sigma_{ps,t} \le \bar{x}\sqrt{V} \end{aligned}$$

With respect to the risk constraint the consumer faces, we derive his asset demand in the following way

$$x_t = \begin{cases} \frac{\pi_t}{\alpha^2 V} - \frac{1}{\pi_t}\left((1-\tau)\; Re + R_{ps,t}\tau e\right) & \text{if} \quad q_{t-1}^e > q_{t-1}^{e,crit} \\ \bar{x} - \frac{\tau e}{\sqrt{V}}\sigma_{ps,t} & \text{if} \quad q_{t-1}^e \le q_{t-1}^{e,crit} \\ 0 & \text{if} \quad \tau e \sigma_{ps,t} \ge \bar{x}\sqrt{V} \end{cases}$$

The funded pension system is endowed with a utility function with constant absolute risk aversion, which leads to an asset demand of the following classical form.

$$\tilde{x}_t = \frac{1}{\tilde{\alpha} V}\,\tilde{\pi}_t$$

We assume that the pension system has unbiased predictions for future cum-dividend prices. Therefore the following must hold true in every period t

$$p_t \overset{!}{=} \tilde{q}_{t-1}^e - \mathbb{E}_{t-1} d_t$$

As we observe three different types of asset demand by the consumer, depending on the critical value $q_{t-1}^{e,crit}$, we derive three different types of equilibrium prices. Hence, the unbiased prediction of the pension system has three peculiarities.

$$\tilde{q}_t^e = \begin{cases} R\,\left(\tilde{q}_{t-1}^e - \mathbb{E}_{t-1}\, d_t\right) + \tilde{\alpha}\, V \bar{x} - 2\frac{\tilde{\alpha}}{\alpha^2} q_t^e + \frac{1}{q_t^e + R(\mathbb{E}_{t-1} d_t - \tilde{q}_{t-1}^e)} A \\ R\,\left(\tilde{q}_{t-1}^e - \mathbb{E}_{t-1}\, d_t\right) + \tilde{\alpha}\,\sqrt{V}\tau\, e\, \sigma_{ps,t} \\ R\,\left(\tilde{q}_{t-1}^e - \mathbb{E}_{t-1}\, d_t\right) + \tilde{\alpha}\, V \bar{x} \end{cases}$$

We can define the old-age consumption of a consumer born in $t-1$ in the following way:

$$\tilde{C}_t = R\, e + (p_t + d_t - R p_{t-1})\, \bar{x}$$

Theorem 4.1 *If a funded pension system exists and the consumer is endowed with an utility function, which features some income effects, an enhancement of the tax rate can have a welfare improving effect, depending on the underlying structure.*

1. $\mathbf{q_{t-1}^e > q_{t-1}^{e,crit}}$

$$\frac{\partial \tilde{C}_t}{\partial \tau} = \begin{cases} > 0 \text{ if } R_{ps,t-1} - R > 0 \\ < 0 \text{ if } R_{ps,t-1} - R < 0 \end{cases}$$

2. $\mathbf{q_{t-1}^e \le q_{t-1}^{e,crit}}$

$$\frac{\partial \tilde{C}_t}{\partial \tau} = \tilde{\alpha}\sqrt{V} e \sigma_{ps,t-1}\, \bar{x}\; > 0$$

3. $\mathbf{\tau\, e\, \sigma_{ps,t} \ge \bar{x}\sqrt{V}}$

$$\frac{\partial \tilde{C}_t}{\partial \tau} = 0$$

This shows only the effects on the welfare if the tax rate is enhanced. The question, which is still open, is whether an introduction of a funded pension is welfare improving or not.

Theorem 4.2 *In the above discussed economy, the introduction of a funded pension system can improve the welfare of the consumer.*

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