Generalized adaptive expectations dynamics in two-stage games

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Abstract

A non-cooperative, two-stage-duopoly game is introduced where firms choose actions in both stages according to best replies with respect to generalized adaptive expectations about the opponents actions in both stages. When building expectations, firms take into account that opponents use best replies based on their expectations about the firms own action. These expectations might again be based on anticipated best-reply behavior of the firm. Iterating this kind of reasoning introduces a general degree of adaptive order, where we distinguish between the first- and second-stage expectations. In the framework of a differentiated Cournot duopoly with R&D competition, we show that for an adaptive order smaller than two the steady state of the best-reply dynamics coincides with the Nash-equilibrium of a version of the game where stage one actions are not revealed before actions in the second stage are chosen. Furthermore, it is demonstrated that if the adaptive order goes to infinity the steady state of the system converges to the sub-game perfect equilibrium of the static game where first stage actions are common knowledge at stage two. In this way the paper provides an innovative adaptive foundation of the transition from the Nash-equilibrium of the corresponding simultaneous move game to the sub-game-perfect equilibrium of the two-stage game.

1. Motivation

The underlying basic model was introduced by Qiu [1]. There a two-stage game is considered with two firms producing differentiated goods in a Cournot duopoly. In the first stage each firm undertakes cost-reducing R&D efforts, and in the second stage the firms produce and sell their products in the market. The inverse demand function is given by $p_i = a - q_i - \gamma q_j$ for $\gamma \in (0,1)$. By spending the R&D efforts the firms can lower the marginal costs, given by[1] $c_i = c - x_i$ and the R&D expenditure function of firm i is quadratic, i.e. $V(x_i) = \frac{vx_i^2}{2}$. The sub-game perfect equilibrium is derived and denoted by (x^*_{SP}, q^*_{SP}).

2. Bounded rational firms

At each stage, each firm is building expectations about present and future actions given current information. As stressed by many authors, R&D investments are often considered as investments in a knowledge capital stock created by firms, that is, their gradually accumulated knowledge as a result of past efforts. In our context the efforts can be seen as search and bargaining costs associated with the determination of prices of production inputs. Therefore, we introduce a general cost reducing investment (CRI) with no cumulative property. The profit maximization with respect to the quantity in the market stage yields a best reply of the form

$$q_{i,t+1} = R_i^q\big(x_{i,t+1}, \mathbb{E}_i^2 q_{j,t+1}\big). \tag{1}$$

$\mathbb{E}_i^2 q_{j,t+1}$ denotes firm i's expectation in the second stage about the competitors output after having observed the CRI in the first stage. In order to derive the best reply on the first stage, the firms maximize $\pi_i^x(x_{i,t+1}, q_{i,t+1}, \mathbb{E}_i^1 q_{j,t+1})$ knowing that it will choose its own quantity by using the best reply (1) of the second stage but now with different expectations, i.e. $\mathbb{E}_i^1 q_{j,t+1}$. For this reason the maximization problem yields

$$\max_{x_{i,t+1}} \pi_i^x\big(x_{i,t+1}, R_i^q(x_{i,t+1}, \mathbb{E}_i^1 q_{j,t+1}), \mathbb{E}_i^1 q_{j,t+1}\big) \tag{2}$$

which results in the best reply of the form

$$x_{i,t+1} = R_i^x(\mathbb{E}_i^1 q_{j,t+1}). \tag{3}$$

3. AE(1)-process

The bounded rational firms use the adaptive expectations rule in both stages. In the second stage, this suggests to adjust the firm i's expectation by

$$\mathbb{E}_i^2 q_{j,t+1} = \alpha q_{j,t} + (1-\alpha) R_j^q\big(x_{j,t+1}, \mathbb{E}_i^2 \mathbb{E}_j^2 q_{i,t+1}\big) \tag{4}$$

with $\mathbb{E}_i^2 \mathbb{E}_j^2 q_{i,t+1}$ being the expectation that firm i expects firm j to expect that firm i itself will produce. Profit maximization leads to map $\tilde{R}_i^{q,1} : \mathbb{R}_+^4 \to \mathbb{R}_+$, given by

$$q_{i,t+1} = \tilde{R}_i^{q,1}\big(x_{i,t+1}, x_{j,t+1}, \mathbb{E}_i^2 \mathbb{E}_j^2 q_{i,t+1}, q_{j,t}\big). \tag{5}$$

In order the describe the first stage, some notations are necessary. Formally, the quantity that firm i expects on the first stage that firm j expects on the first stage, that firm i will produce, is given by $\mathbb{E}_i^1 \mathbb{E}_j^1 q_{i,t+1}$, whereas $\mathbb{E}_i^1 \mathbb{E}_j^2 q_{i,t+1}$ defines the quantity that firm i expects on the first stage that firm j expects on the second stage, that firm i will produce. The adaptive rule with respect to the quantity yields

$$\mathbb{E}_i^2 q_{j,t+1} = \alpha q_{j,t} + (1-\alpha) R_j^q\big(\mathbb{E}_i^1 x_{j,t+1}, \mathbb{E}_i^1 \mathbb{E}_j^2 q_{i,t+1}\big) \tag{6}$$

where $\mathbb{E}_i^1 x_{j,t+1}$ is also adjusted using adaptively, i.e.

$$\mathbb{E}_i^1 x_{j,t+1} = \alpha x_{j,t} + (1-\alpha) R_j^x\big(\mathbb{E}_i^1 \mathbb{E}_j^1 q_{i,t+1}\big). \tag{7}$$

Profit maximization leads to a map $\tilde{R}_i^{x,1} : \mathbb{R}_+^4 \to \mathbb{R}_+$, given by

$$x_{i,t+1} = \tilde{R}_i^{x,1}\big(x_{j,t}, q_{j,t}, \mathbb{E}_i^1 \mathbb{E}_j^1 q_{i,t+1}, \mathbb{E}_i^1 \mathbb{E}_j^2 q_{i,t+1}\big) \tag{8}$$

Combining (5) and (8) and assuming $\mathbb{E}_i^1 \mathbb{E}_j^1 q_{i,t+1} = \mathbb{E}_i^1 \mathbb{E}_j^2 q_{i,t+1} = q_{i,t}$ yields a unique fixed point $(x^*_{AE(1)}, q^*_{AE(1)})$ which is independent of α, meaning that it coincides also with the fixed point of the best reply dynamics. Furthermore this fixed point coincides with the Nash-equilibrium of a simultaneous move version of this game.

4. AE(2)-process

This process exhibits a strategic effect on the first stage, because the expectation iteration of order two changes the profit maximization problem with respect to the CRI. On the second stage, there is never a strategic effect. Formally, the adaptive expectation process on the market stage is now used again in order to describe $\mathbb{E}_i^2 \mathbb{E}_j^2 q_{i,t+1}$ in (4) and is given by

$$\mathbb{E}_i^2 \mathbb{E}_j^2 q_{i,t+1} = \alpha q_{i,t} + (1-\alpha) R_i^q\big(x_{i,t+1}, \mathbb{E}_i^2 \mathbb{E}_j^2 \mathbb{E}_i^2 q_{j,t+1}\big).$$

On the first stage the expectation formation according to the CRI has to be revisited again. In the AE(1)-process, given by (6), $\mathbb{E}_i^1 \mathbb{E}_j^2 q_{i,t+1}$ has to be substituted in order to get an AE(2)-process, i.e.

$$\mathbb{E}_i^1 \mathbb{E}_j^2 q_{i,t+1} = \alpha q_{i,t} + (1-\alpha) R_i^q\big(\mathbb{E}_i^1 \mathbb{E}_j^2 x_{i,t+1}, \mathbb{E}_i^1 \mathbb{E}_j^1 \mathbb{E}_i^2 q_{j,t+1}\big)$$

From firm i's point of view, $\mathbb{E}_i^1 \mathbb{E}_j^2 x_{i,t+1}$ represents the CRI, that firm i expects on the first stage, that firm j expects on the second stage about $x_{i,t+1}$. But at the second stage $x_{i,t+1}$ is observable for firm j and therefore the following notation makes sense

$$\mathbb{E}_i^1 \mathbb{E}_j^2 x_{i,t+1} = x_{i,t+1}. \tag{9}$$

This influences the profit maximization problem and after replacing also $\mathbb{E}_i^1 x_{j,t+1}$ in (6) by the adaptive rule[2] and deriving the best reply we get the conclusion:

Proposition 4.1 *For all $\gamma \in [0,1]$, $\alpha \in [0,1]$ and $v > \frac{(4+(\alpha-1)^2\gamma^2)^2}{32}$ a unique positive fixed point*

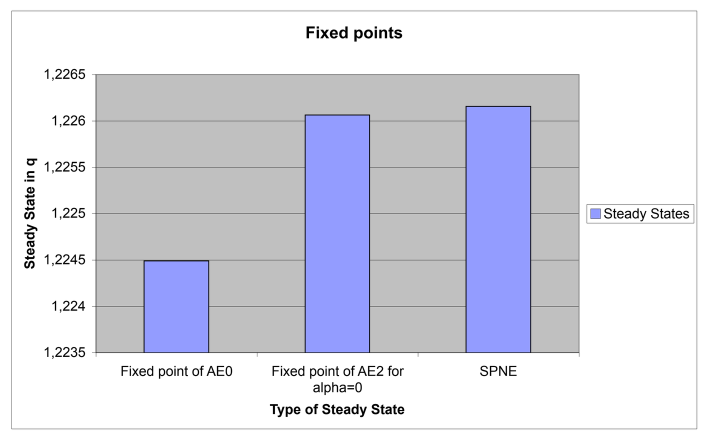
$$E_{AE(2)}(\alpha) = (x^*_{AE(2)}(\alpha), x^*_{AE(2)}(\alpha), q^*_{AE(2)}(\alpha), q^*_{AE(2)}(\alpha))$$

with $x^*_{AE(2)} : [0,1] \to [x^*_{AE(1)}, x^*_{SP})$

and $q^*_{AE(2)} : [0,1] \to [q^*_{AE(1)}, q^*_{SP})$

exists with $\frac{\partial x^*_{AE(2)}}{\partial \alpha} < 0$ and $\frac{\partial q^*_{AE(2)}}{\partial \alpha} < 0$.

Figure 1: *For $\gamma = .5$, $a = 5$, $c = 2$, $v = 20$: $q_{AE(1)}, q^*_{AE(2)}(0)$ and q^*_{SP}*



5. AE(n)-process

We assume that in an AE(k)-process, the firms are able to build expectations of order $k+1$ for all $k \in \mathbb{N}$. Define[3]

$$\lim_{n\to\infty} x^*_{AE(n)}(\alpha) =: x^*(\alpha)$$
$$\lim_{n\to\infty} q^*_{AE(n)}(\alpha) =: q^*(\alpha) \tag{10}$$

Then $x^*(\alpha) \in [x^*_{NE}, x^*_{SP}]$ and $q^*(\alpha) \in [q^*_{NE}, q^*_{SP}]$. By only considering iterated best reply dynamics of infinite order, we obtain

$$\lim_{n\to\infty} x^*_{AE(n)}(0) = x^*_{SP}$$
$$\lim_{n\to\infty} q^*_{AE(n)}(0) = q^*_{SP}.$$

References

[1] Larry D. Qiu, *On the dynamic efficiency of bertrand and cournot equilibria*, Journal of Economic Theory **75** (1997), 213–229.

[1] In the original paper also spillover effects occur.

[2] Details are omitted here.

[3] These functions are decreasing in α and formally we distinguish between n odd and even because only for n even a new strategic effect occurs.